Math 42 Midterm 3 Review

You will need a calculator to solve the problems marked ★.
You should **NOT** use a calculator for any other problems.

[0] Write out all the basic identities:

[a] $\sin(-x) =$ $\qquad$ $\cos(-x) =$ $\qquad$ $\tan(-x) =$

  $\sec(-x) =$ $\qquad$ $\csc(-x) =$ $\qquad$ $\cot(-x) =$

[b] $\sin x = \dfrac{1}{\quad\quad}$ $\qquad$ $\cos x = \dfrac{1}{\quad\quad}$ $\qquad$ $\tan x = \dfrac{1}{\quad\quad}$

  $\sec x = \dfrac{1}{\quad\quad}$ $\qquad$ $\csc x = \dfrac{1}{\quad\quad}$ $\qquad$ $\cot x = \dfrac{1}{\quad\quad}$

[c] $\tan x = \dfrac{x}{x}$ $\qquad$ $\cot x = \dfrac{x}{x}$

[d] $^2x + \quad ^2x = 1$ $\qquad$ $^2x - \quad ^2x = 1$ **2 versions**

  $1 + \quad ^2x = \quad ^2x$ **2 versions** $\qquad$ $1 - \quad ^2x = \quad ^2x$ **2 versions** $\qquad$ $^2x - 1 = \quad ^2x$ **2 versions**

[e] $\sin(x + y) =$ $\qquad$ $\cos(x + y) =$ $\qquad$ $\tan(x + y) =$

  $\sin(x - y) =$ $\qquad$ $\cos(x - y) =$ $\qquad$ $\tan(x - y) =$

[f] $\sin 2x =$ $\qquad$ $\cos 2x =$ **3 versions** $\tan 2x =$

[g] $\sin^2 x =$ $\qquad$ $\cos^2 x =$ $\qquad$ **These answers should not contain the square of any function**

[h] $\sin \frac{1}{2} x =$ $\qquad$ $\cos \frac{1}{2} x =$ $\qquad$ $\tan \frac{1}{2} x =$ **4 versions**

[00] Prove the most difficult looking identities in the textbook exercises from sections 5.2, 5.4, 5.5, chapter 5 review, chapter 5 test and additional homework assignment #2.

[1]   Simplify the following expressions completely.
        A product of two factors is simpler than a sum of two terms.
        There should be as few terms, factors and trigonometric functions as possible.
        Exponents should be as small as possible (without using negative exponents).

[a] $\sec x \cot x \sin x$

[b] $(1 - \sec x)(1 + \cos x)$

[c] $\dfrac{\sec x}{\csc x} + \dfrac{\cos x}{\sin x}$

[d] $2\cot 2x - \cot x + \tan x$

[e] $6\cos \frac{7\pi}{12} \sin \frac{7\pi}{12}$

[f] $5 - 10\cos^2 \frac{3\pi}{8}$

[g] $\sin^2 165°$

[h] $\dfrac{\tan^2 75° - 1}{\tan 75°}$

[i] $\cos 105° \sin 15° + \sin 105° \cos 15°$

[j] $\sin \frac{\pi}{4} \sin \frac{11\pi}{12} + \cos \frac{\pi}{4} \cos \frac{11\pi}{12}$

[2]   If $\sec x = -\frac{7}{5}$ and $\pi < x < \frac{3\pi}{2}$, and $\cot y = -\frac{3}{4}$ and $\sin y > 0$, find the values of the following expressions.
        Simplify your final answers.

[a] $\sin 2y$

[b] $\tan(x - y)$

[c] $\cos \frac{x}{2}$

[d] $\cos(x - \frac{5\pi}{3})$

[e] $\csc \frac{x}{2}$

[f] $\tan 2x$

[g] $\sin(y + \arccos(-\frac{1}{5}))$

[h] $\sec(x + y)$

[i] $\cos 2y$

[3]   Use the power reducing formulae to rewrite the following expressions using only the first powers of cosine.
        Your final answers must **NOT** be in factored form. Simplify your final answers.

[a] $\sin^4 x$

[b] $\sin^2 x \cos^4 x$

[4]   Rewrite the following expressions using only $\sin x$, $\cos x$ and/or $\tan x$.
        Simplify your final answers to use as few trigonometric functions as possible.

[a] $\sin 3x$

[b] $\tan 4x$

[c] $\cos 4x$

[5]   Find the value of the following expressions. Simplify your final answers.

[a] $\cos(\arctan(-2) - \sin^{-1}(-\frac{3}{7}))$

[b] $\sin(2\tan^{-1} 3)$

[c] $\tan(\sin^{-1} \frac{1}{3} - \arccos(-\frac{2}{3}))$

[6]   Solve the following equations.

[a] $\tan x + 3\sqrt{3} = 2(\sqrt{3} - \tan x)$

[b] $5(1 + \csc x) = 4 - 3\cot^2 x$

[c] $4\cos 4x + 2\sqrt{2} = 0$

[d] $\sqrt{3} \cot \frac{1}{3} x - 3 = 0$

[e] $\sin 2x + \sqrt{3} \cos x = 0$

[f] $3\cos \frac{1}{2} x - 2\cos x = 1$ ★